A Statistical Framework for Real-time Event Detection in Power Systems

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Abstract

A quickest change detection (QCD) algorithm is applied to the problem of detecting and identifying line outages in a power system. The statistics of electricity demand are assumed to be known and propagated through a linearized model of the equations describing the power flow balance. The voltage phase angle measurements are collected using phasor measurement units (PMUs). It is shown that the proposed algorithm is applicable to situations where the transient dynamics of the power system following a line outage are considered. Case studies demonstrating the performance of the QCD algorithm to faults with transient dynamics are illustrated through the IEEE 14-bus test system.

I. Introduction

Timely detection of line outages in a power system is crucial for maintaining operational reliability. Toward this end, many online decision-making tools rely on a system model that is obtained offline, which can be inaccurate due to bad historical or telemetry data. Such inaccuracies have been a contributing factor in many recent blackouts. For example, in the 2011 San Diego blackout, operators were unable to determine overloaded lines because the network model was not up to date [1]. This lack of situational awareness limited the ability of the operators to identify and prevent the next critical contingency, and led to a cascading failure. Similarly, during the 2003 US Northeastern blackout, operators failed to initiate the correct remedial schemes because they had an inaccurate model of the power system and could not identify the loss of key transmission elements [2]. These blackouts highlight the importance of developing online measurement-based techniques to detect and identify system topological changes that arise from line outages. In this paper, we tackle such topology change detection problems by utilizing measurements provided by phasor measurement units (PMUs).

The authors of [3] developed a method for line outage detection and identification based on the theory of quickest change detection (QCD) [4], [5]. In this method, the incremental changes in real power injections are modeled as independent zero-mean Gaussian random variables. Then, the probability distribution of such incremental changes is mapped to that of the incremental changes in voltage phase angles via a linear transformation obtained from the power flow balance equations. The PMUs provide a random sequence of voltage phase angle measurements in real-time; when a line outage occurs, the probability distribution of the incremental changes in the voltage phase angles changes abruptly. The objective is to detect a change in this probability distribution after the occurrence of a line outage as quickly as possible while maintaining a desired false alarm rate.

This paper extends the method proposed in [3] by considering the power system transient response immediately following the line outage. For example, after an outage, the transient behavior of the system is dominated by the inertial response from the generators. This is followed by the governor response and then the automatic generation control (AGC). We incorporate these dynamics into the power system model by relating incremental changes in active power demand to active power generation. We use this model to develop the Dynamic CuSum test (D-CuSum), which is used to capture the transient behavior in the non-composite QCD problem. Then, the Generalized Dynamic CuSum test (G-D-CuSum) is derived by calculating a D-CuSum statistic for each possible line outage scenario; an outage is declared the first time any of the test statistics crosses a pre-specified threshold. The proposed test has better performance because it takes the transient behavior into account in addition to the persistent change in the distribution that results from the outage.

The remainder of this paper is organized as follows. In Section II, we describe the model of the power system adopted in this work, and introduce the statistics describing the voltage phase angle before, during, and after the occurrence of an outage. In Section III, the proposed QCD-based line outage identification algorithm that accounts for transient behavior after a line outage is introduced. In Section IV, we illustrate the proposed ideas via numerical case studies on the IEEE 14-bus test system. Finally, concluding remarks and directions for future work are provided in Section V.

II. Power System Model

Let $\mathcal{L} = \{1, \ldots, L\}$ denote the set of lines in the system. At time t, let $V_i(t)$ and $\theta_i(t)$ denote the voltage magnitude and phase angle at bus i respectively, and let $P_i(t)$ and $Q_i(t)$ denote the net active and reactive power injection at bus i, respectively. Then, the quasi-steady-state behavior of the system can be described by the power flow equations, which for bus i can be compactly written as:

$$\begin{aligned} P_i(t) &= p_i(\theta_1(t), \ldots, \theta_N(t), V_1(t), \ldots, V_N(t)), \\ Q_i(t) &= q_i(\theta_1(t), \ldots, \theta_N(t), V_1(t), \ldots, V_N(t)), \end{aligned} \tag{1}$$

where the dependence on the system network parameters is implicitly captured by $p_i(\cdot)$ and $q_i(\cdot)$. The outage of line $\ell \in \mathcal{L}$ at time $t = t_f$ is assumed to be persistent (i.e., the line is not restored until it is detected to be outaged), with $\gamma_0 \Delta t \leq t_f < (\gamma_0 + 1)\Delta t$, where Δt is the time between successive PMU samples. In addition, assume that the loss of line ℓ does not cause islands to form in the post-event system (i.e., the underlying graph representing the internal power system remains connected).

A. Pre-outage Model

Let $P_i[k] := P_i(k\Delta t)$ and $Q_i[k] := Q_i(k\Delta t)$, $\Delta t > 0$, $k = 0, 1, 2, \ldots$, denote the k^{th} measurement sample of active and reactive power injections into bus i. Similarly, let $V_i[k]$ and $\theta_i[k]$, $k = 0, 1, 2, \ldots$, denote bus i's k^{th} voltage magnitude and angle measurement sample. Furthermore, define variations in voltage magnitudes and phase angles between consecutive sampling times $k\Delta t$ and $(k+1)\Delta t$ as $\Delta V_i[k] := V_i[k+1] - V_i[k]$, and $\Delta\theta_i[k] := \theta_i[k+1] - \theta_i[k]$, respectively. Similarly, variations in the active and reactive power injections at bus i between two consecutive sampling times are defined as $\Delta P_i[k] = P_i[k+1] - P_i[k]$ and $\Delta Q_i[k] = Q_i[k+1] - Q_i[k]$.

Proceeding in the same manner as in [3], we linearize (1) around $(\theta_i[k], V_i[k], P_i[k], Q_i[k])$, $i = 1, \ldots, N$, and use the DC power flow assumptions to decouple the real and reactive power flow equations. Then, after omitting the equation corresponding to the reference bus, the relationship between voltage phase angles and the variations in the real power injection can be expressed as:

$$\Delta P[k] \approx H_0 \Delta\theta[k], \tag{2}$$

where $\Delta P[k]$, $\Delta\theta[k] \in \mathbb{R}^{(N-1)}$ and $H_0 \in \mathbb{R}^{(N-1)\times(N-1)}$ is the imaginary part of the system admittance matrix with the row and column corresponding to the reference bus removed.

In an actual power system, random fluctuations in the load drive the generator response. Therefore, in this paper, we use the so-called governor power flow model (see e.g., [6]), which is more realistic than the conventional power flow model, where the slack bus picks up any changes in the load power demand. In the governor power flow model, at time instant k, the relation between changes in the load demand vector, $\Delta P^d[k] \in \mathbb{R}^{N_d}$, and changes in the power generation vector, $\Delta P^g[k] \in \mathbb{R}^{N_g}$, is described by

$$\Delta P^g[k] = B(t)\Delta P^d[k], \tag{3}$$

where $B(t)$ is a time dependent matrix of participation factors. We approximate $B(t)$ by quantizing it to take values B_i, $i = 0, 1, \ldots, T$, where i denotes the time period of interest. Let $B(t) = B_0$ and $M_0 := H_0^{-1}$

during the pre-outage period. Then, we can substitute (3) into (2) to obtain a pre-outage relation between the changes in the voltage angles and the real power demand at the load buses as follows:

$$\begin{aligned}\Delta\theta[k] &\approx M_0 \Delta P[k] \\ &= M_0 \begin{bmatrix} \Delta P^g[k] \\ \Delta P^d[k] \end{bmatrix} \\ &= [M_0^1 \ M_0^2] \begin{bmatrix} B_0 \Delta P^d[k] \\ \Delta P^d[k] \end{bmatrix} \\ &= (M_0^1 B_0 + M_0^2)\Delta P^d[k] \\ &= \tilde{M}_0 \Delta P^d[k],\end{aligned} \tag{4}$$

where $\tilde{M}_0 = M_0^1 B_0 + M_0^2$.

B. Instantaneous Change During Outage

At the time of outage, $t = t_f$, there is an instantaneous change in the mean of the voltage phase angle measurements that affects only one incremental sample, namely, $\Delta\theta[\gamma_0] = \theta[\gamma_0 + 1] - \theta[\gamma_0]$. The measurement $\theta[\gamma_0]$ is taken immediately prior to the outage, whereas $\theta[\gamma_0 + 1]$ is the measurement taken immediately after the outage. Suppose the outaged line ℓ connects buses m and n. Then, the effect of an outage in line ℓ can be modeled with a power injection of $P_\ell[\gamma_0]$ at bus m and $-P_\ell[\gamma_0]$ at bus n, where $P_\ell[\gamma_0]$ is the pre-outage line flow across line ℓ from m to n. Following a similar approach as the one in [3], the relation between the incremental voltage phase angle at the instant of outage, $\Delta\theta[\gamma_0]$, and the variations in the real power flow can be expressed as:

$$\Delta\theta[\gamma_0] \approx M_0 \Delta P[\gamma_0] - P_\ell[\gamma_0 + 1] M_0 r_\ell, \tag{5}$$

where $r_\ell \in \mathbb{R}^{N-1}$ is a vector with the $(m-1)^{\text{th}}$ entry equal to 1, the $(n-1)^{\text{th}}$ entry equal to -1, and all other entries equal to 0. Furthermore, by using the governor power flow model of (3) and substituting into (5), and simplifying, we obtain:

$$\Delta\theta[\gamma_0] \approx \tilde{M}_0 \Delta P^d[\gamma_0] - P_\ell[\gamma_0 + 1] M_0 r_\ell. \tag{6}$$

C. Post-Outage

Following a line outage, the power system undergoes a transient response governed by B_i, $i = 1, 2, \ldots, T-1$ until quasi-steadystate is reached, in which $B(t)$ settles to a constant B_T. For example, immediately after the outage occurs, the power system is dominated by the inertial response of the generators, which is then followed by the governor response. As a result of the line outage, the system topology changes, which manifests itself in the matrix H_0. This change in the matrix H_0 resulting from the outage can be expressed as the sum of the pre-outage matrix and a perturbation matrix, ΔH_ℓ, i.e., $H_\ell = H_0 + \Delta H_\ell$. Then, by letting $M_\ell := H_\ell^{-1} = [M_\ell^1 \ M_\ell^2]$, and deriving in the same manner as the pre-outage model of (4), we obtain the post-outage relation between the changes in the voltage angles and the real power demand as:

$$\Delta\theta[k] \approx \tilde{M}_{\ell,i} \Delta P^d[k], \quad \gamma_{i-1} \le k < \gamma_i, \tag{7}$$

where $\tilde{M}_{\ell,i} = M_\ell^1 B_i + M_\ell^2$, $i = 1, 2, \ldots, T$.

D. Measurement Model

Since the voltage phase angles, $\theta[k]$, are assumed to be measured by PMUs, we allow for the scenario where the angles are measured at only a subset of the load buses, and denote this reduced measurement set by $\hat{\theta}[k]$. Suppose that there are N_d load buses and we select $p \leq N_d$ locations to deploy the PMUs. Then, there are $\binom{N_d}{p}$ possible locations to place the PMUs. In this paper, we assume that the PMU locations are fixed; in general, the problem of optimal PMU placement is NP-hard and its treatment is beyond the scope of this paper.

Let

$$\tilde{M} = \begin{cases} \tilde{M}_0, \text{ if } 1 \leq k < \gamma_0, \\ \vdots \\ \tilde{M}_{\ell,K}, \text{ if } k \geq \gamma_{T-1}. \end{cases} \tag{8}$$

Then, the absence of a PMU at bus i corresponds to removing the i^{th} row of $\tilde{M}$. Thus, let $\hat{M} \in \mathbb{R}^{p \times N_d}$ be the matrix obtained by removing $N-p-1$ rows from $\tilde{M}$. Therefore, we can relate $\hat{M}$ to $\tilde{M}$ in (8) as follows:

$$\hat{M} = C\tilde{M}, \tag{9}$$

where $C \in \mathbb{R}^{p \times (N-1)}$ is a matrix of 1's and 0's that appropriately selects the rows of $\tilde{M}$. Accordingly, the increments in the phase angle can be expressed as follows:

$$\Delta\hat{\theta}[k] \approx \hat{M}\Delta P^d[k]. \tag{10}$$

The small variations in the real power injections at the load buses, $\Delta P^d[k]$, can be attributed to random fluctuations in electricity consumption. In this regard, we may model the $\Delta P^d[k]$'s as independent and identically distributed (i.i.d.) random vectors. By the Central Limit Theorem [7], it can be shown that each $\Delta P^d[k]$ is a Gaussian vector, i.e., $\Delta P^d[k] \sim \mathcal{N}(0, \Lambda)$, where Λ is the covariance matrix. Note that the elements $\Delta P^d[k]$ need not be independent. Since $\Delta\hat{\theta}[k]$ depends on $\Delta P^d[k]$ through the linear relationship given in (10), we have that:

$$\Delta\hat{\theta}[k] \sim \begin{cases} f_0 := \mathcal{N}(0, \hat{M}_0\Lambda\hat{M}_0^T), \text{ if } 1 \leq k < \gamma_0, \\ f_\ell^{(0)} := \mathcal{N}(-P_\ell[\gamma+1]CM_0 r_\ell, \\ \qquad\qquad \hat{M}_0\Lambda\hat{M}_0^T), \text{ if } k = \gamma_0, \\ \vdots \\ f_\ell^{(T)} := \mathcal{N}(0, \hat{M}_{\ell,K}\Lambda\hat{M}_{\ell,K}^T), \text{ if } k \geq \gamma_{T-1}, \end{cases} \tag{11}$$

It is important to note that for $\mathcal{N}\left(0, \hat{M}\Lambda\hat{M}^T\right)$ to be a nondegenerate p.d.f., its covariance matrix, $\hat{M}\Lambda\hat{M}^T$, must be full rank. We enforce this by ensuring that the number of PMUs allocated, p, is less than or equal to the number of load buses, N_d.

III. Line Outage Detection Using QCD

In the line outage detection problem setting, the goal is to detect the outage in line ℓ as quickly as possible subject to false alarm constraints. The outage induces a change in the statistical characteristics of the observed sequence $\{\Delta\hat{\theta}[k]\}_{k\geq 1}$. The aim is to design stopping rules that detect this change. A stopping time τ, adapted to the observed sequence, is a random time during which a line outage is declared.

A. Problem Setup

The goal in QCD is to design stopping rules that will detect the change in the statistical behavior of the observed process as fast as possible under false alarm constraints. The false alarm constraint that we choose is based on the mean time to false alarm; thus, we would like $\mathbb{E}_\infty[\tau] \geq \beta$, where $\beta > 0$ is a pre-determined parameter, and $\mathbb{E}_\infty$ is the expectation under the probability measure where no outage has occurred.

In order to quantify the detection delay for line outages, we introduce the following delay metric:

$$\mathsf{D}_\ell(\tau) = \sup_{\gamma_0 \geq 1} \operatorname{ess\,sup} \mathbb{E}_{\gamma_0,\ell}\left[(\tau - \gamma_0)^+ \middle| \Delta\hat{\theta}[1], \ldots, \Delta\hat{\theta}[\gamma_0 - 1]\right]. \tag{12}$$

According to (12), the delay of stopping time τ for detecting an outage in line ℓ is measured by taking the expected value of $(\tau - \gamma_0)^+$ under the assumption that (i) the underlying distribution is the one induced on the observations when an outage occurs in line ℓ at time instant γ_0, and (ii) conditioning on a set of pre-outage observations $\{\Delta\hat{\theta}[1], \ldots, \Delta\hat{\theta}[\gamma_0 - 1]\}$.

In the setting described in Section II, we assume that a sequence of observations $\{\Delta\hat{\theta}[k]\}_{k\geq 1}$ is measured by PMUs and passed sequentially to a decision maker. According to the statistical model in (11), before an outage has occurred, $\Delta\hat{\theta}[k] \sim f_0$. At an unknown time instant γ_0, an outage occurs in line ℓ and the distribution of $\Delta\hat{\theta}[k]$ changes from f_0 to $f_\ell^{(0)}$. Then, the system undergoes a series of transient responses which corresponds to the distribution of $\Delta\hat{\theta}[k]$ evolving from $f_\ell^{(0)}$ to $f_\ell^{(T)}$. First, a meanshift takes place during the instant of change γ_0, where the pdf is $f_\ell^{(0)}$. Then, the statistical behavior of the process is characterized by a series of changes only in the covariance matrix of the measurements.

B. Generalized CuSum Test

The Generalized CuSum (G-CuSum) based test was proposed as a line outage detection scheme in [8] with the understanding that the transition between pre- and post-outage periods is not characterized by any transient behavior other than the meanshift that occurs at the instant of outage. The meanshift was captured by introducing an additional log-likelihood ratio term between the distribution at the time of change and the distribution before the change. The final test statistic takes the maximum of this log-likelihood ratio and the traditional G-CuSum test recursion.

Although, the G-CuSum algorithm does not take any transient dynamics into consideration, it can still perform well when the transient distributions and the final post- change distribution are "similar", i.e., when the KL divergence between $f_\ell^{(i)}$, $i = 1, 2, \ldots, T-1$, and $f_\ell^{(T)}$ is small. As a result, it is insightful to compare the performance of the G-CuSum test with the performance of the G-D-CuSum test that is proposed in this work.

Since the line that is outaged is not known, the G-CuSum test works by using the CuSum test statistics in a generalized manner. As a result, we compute L CuSum statistics in parallel, one corresponding to each line outage scenario, and declare a change when an outage to any line is detected. The CuSum recursion for line ℓ is calculated by accumulating log-likelihood ratios between $f_\ell^{(T)}$ and f_0. In particular, define the G-CuSum statistic corresponding to line ℓ outage recursively as:

$$\begin{aligned} W_\ell^{\mathrm{C}}[k] = \max\Bigg\{ & W_\ell^{\mathrm{C}}[k-1] + \log \frac{f_\ell^{(T)}(\Delta\hat{\theta}[k])}{f_0(\Delta\hat{\theta}[k])}, \\ & \log \frac{f_\ell^{(0)}(\Delta\hat{\theta}[k])}{f_0(\Delta\hat{\theta}[k])}, 0 \Bigg\}, \end{aligned} \tag{13}$$

with $W_\ell^{\mathrm{C}}[0] = 0$ for all $\ell \in \mathcal{L}$.

The goal is to declare an outage as soon as any line is outaged; thus, the algorithm declares a detection the first time any of the line statistics crosses its corresponding threshold. Accordingly, the stopping time of the test is as follows:

$$\tau^{\mathrm{C}} = \inf_{\ell \in \mathcal{E}} \left\{ \inf\{k \geq 1 : W_\ell^{\mathrm{C}}[k] > A_\ell\} \right\}, \tag{14}$$

with $A_\ell > 0$ being the threshold corresponding to line ℓ.

C. Generalized Dynamic CuSum Test

Since the statistical model used in this paper includes an arbitrary number of transient periods with finite duration, each one corresponding to a respective transient distribution induced on the observations, it is clear that the G-CuSum test of [8] needs to be modified to take this transient behavior into consideration. Toward this end, we introduce the Generalized Dynamic CuSum (G-D-CuSum) test. This test is derived by exploiting the so-called Dynamic CuSum (D-CuSum), a test also proposed in this work. This test arises as a solution to the non-composite QCD problem under the presence of an arbitrary number of transient periods. The D-CuSum test statistic is derived by formulating the transient QCD problem as a dynamic composite hypothesis testing problem at each time instant. The G-D-CuSum algorithm uses the test statistics of the D-CuSum test in a generalized manner, i.e., calculates a test statistic for each possible line outage in parallel, and declares an outage when one of the line statistics crosses a pre-determined positive threshold corresponding to the line.

By using the D-CuSum test statistic as a basis, we propose the G-D-CuSum test. The statistic for line ℓ is given as follows:

$$W_\ell^{\mathrm{D}}[k] = \max\left\{ \Omega_\ell^{(1)}[k], \ldots, \Omega_\ell^{(T)}[k], \log \frac{f_\ell^{(0)}(\Delta\hat{\theta}[k])}{f_0(\Delta\hat{\theta}[k])} \right\}, \tag{15}$$

where

$$\Omega_\ell^{(i)}[k] = \max\left\{ \left[\max\{\Omega_\ell^{(i)}[k-1], \Omega_\ell^{(i-1)}[k-1]\} + \log \frac{f_\ell^{(i)}(\Delta\hat{\theta}[k])}{f_0(\Delta\hat{\theta}[k])} \right], 0 \right\}, \tag{16}$$

for $i \in \{1, \ldots, T\}$, $\Omega_\ell^{(0)}[k] := 0$ for all $k \in \mathbb{Z}$ and $\Omega_\ell^{(i)}[0] := 0$ for all $\ell \in \mathcal{L}$ and all i. The corresponding stopping rule is defined as

$$\tau^{\mathrm{D}} = \inf_{\ell \in \mathcal{L}} \left\{ \inf\{k \geq 1 : W_\ell^{\mathrm{D}}[k] > A_\ell\} \right\}. \tag{17}$$

Calculating the test statistic for line ℓ involves calculating the statistics $\Omega_\ell^{(1)}$, …,$\Omega_\ell^{(T)}$. The final test statistic is given by taking the maximum of these terms together with the log-likelihood ratio between the distribution at the outage and the pre-outage distribution. Note that to renew each Ω statistic, the value of the statistic in the previous time instant and the value of the statistic used to detect the previous distribution change is used.

IV. Case Studies

In this section, the algorithm proposed in (15)-(17) is applied to the IEEE 14-bus test system (see [9]), the one-line diagram of which is shown in Fig. 1. In order to compute the transient dynamics following a line outage, we use the simulation tool Power System Toolbox [10]. For simplicity, we used the statistical model in (11) with $T = 2$, i.e., we assumed one transient period after the line outage occurs. Additional transient periods could easily be incorporated into the simulations. The power injection profile at the load buses are assumed to be independent Gaussian random variables with variance of 0.03 and the PMU sampling rate is assumed to be 30 measurements per second.

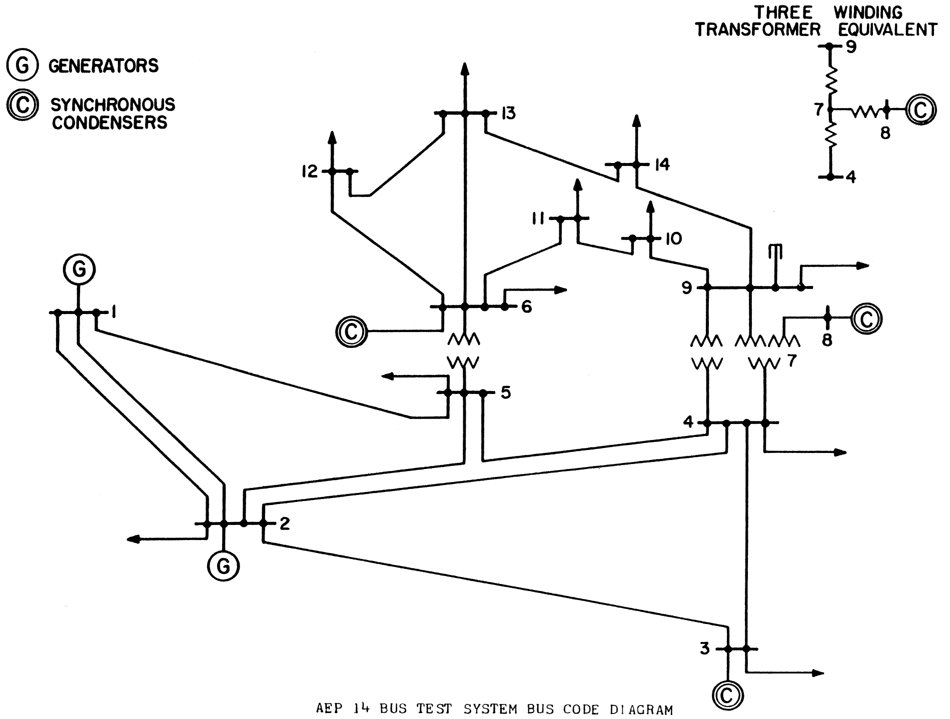

Fig. 1: IEEE 14-bus system one-line diagram.

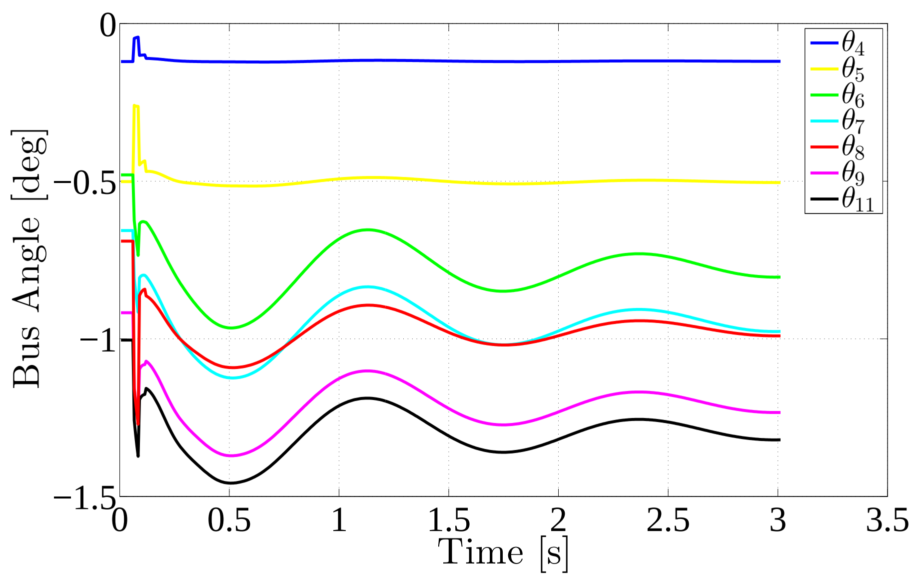

Fig. 2: Voltage phase angles of IEEE 14-bus system following an outage in line 4.

A. 14-bus System

For the IEEE 14-bus system, we simulated an outage in line 4 at $t = 0.1$s. The dynamic responses of the voltage phase angles are shown in Fig. 2. From the plots, we conclude that the transient period following a line outage lasts approximately 3 seconds, which is assumed in the model for our proposed detection algorithm.

We then sample the voltage angles during the transient period and apply the detection algorithm of (15) to the collected data. The threshold A_ℓ is selected to be 400. Starting from $k = 100$, which is when the outage occurs, the W_4^D stream starts to grow while all the other streams W_ℓ^D for $\ell \neq 4$ stay near 0. From the figure, we conclude that at $k = 275$, an outage is declared, which is when $W_4^D > 400$. The detection

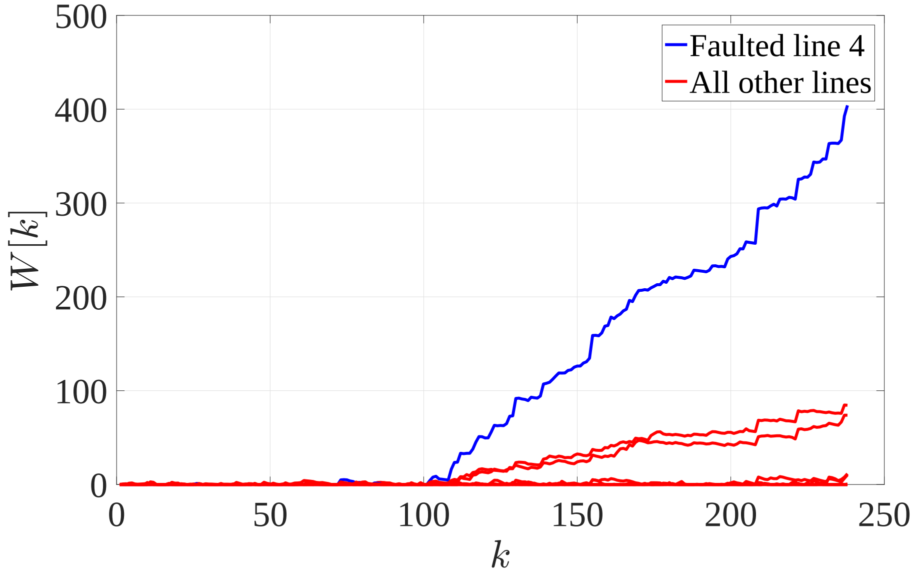

Fig. 3: Voltage phase angles of IEEE 14-bus system following an outage in line 4.

delay is 175 samples.

V. Conclusion

In this paper, we addressed the problem of detecting and identifying line outages by exploiting the statistical properties of voltage phase angle measurements obtained from PMUs in real-time. A relation between the incremental active power demand and active power generation to the voltage phase angles is established through the linearized power balance equations. The system transient response following a line outage is modeled using time dependent participation factors, which more realistically captures the actual system dynamics and results in better detection performance. Simulations are performed on the IEEE 14-bus test systems to illustrate the effectiveness of the proposed line outage detection algorithm.

References

[1] FERC and NERC. (2012, Apr.) Arizona-southern California outages on September 8, 2011: Causes and recommendations. [Online]. Available: http://www.ferc.gov.

[2] U.S.-Canada Power System Outage Task Force. (2004, Apr.) Final report on the August 14th blackout in the United States and Canada: causes and recommendations. [Online]. Available: http://energy.gov.

[3] Y. C. Chen, T. Banerjee, A. D. Domínguez-García, and V. V. Veeravalli, "Quickest line outage detection and identification," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 749–758, 2016.

[4] H. V. Poor and O. Hadjiliadis, *Quickest Detection*. Cambridge University Press, 2009.

[5] V. V. Veeravalli and T. Banerjee, *Quickest Change Detection*. Elsevier: E-reference Signal Processing, 2013.

[6] M. Lotfalian, R. Schlueter, D. Idizior, P. Rusche, S. Tedeschi, L. Shu, and A. Yazdankhah, "Inertial, governor, and AGC/economic dispatch load flow simulations of loss of generation contingencies," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-104, no. 11, pp. 3020 –3028, nov. 1985.

[7] B. Hajek, *Random Processes for Engineers*. Cambridge university press, 2015.

[8] G. Rovatsos, X. Jiang, A. D. Domínguez-García, and V. V. Veeravalli, "Comparison of statistical algorithms for power system line outage detection," in *Proc. of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, Apr. 2016.

[9] "Power system test case archive," Oct. 2012. [Online]. Available: http://www.ee.washington.edu/research/pstca.

[10] J. Chow and K. Cheung, "A toolbox for power system dynamics and control engineering education and research," *IEEE Transactions on Power Systems*, vol. 7, no. 4, pp. 1559–1564, Nov. 1992.