# Power System Fault Location — A Quickest Change Detection Approach

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#### Abstract

Efficacious fault detection, identification, and locationing is critical for the reliable operation of power system networks. This paper describes a method based on the quickest change detection (QCD) algorithm to detect, identify, and locate line outages on transmission systems. A two-level impedance-based method that aims to first detect a fault by detecting a change in the topology of the power system network is applied to data gathered from PMUs. The algorithm then proceeds to locate the fault by identifying a corresponding change in the statistics of the voltage phasors due to a change in the impedance of the faulted line, which is dependent on the fault location. Case studies demonstrating the performance of the proposed algorithm to fault-locationing are illustrated through the IEEE 14-bus test system.

#### I. INTRODUCTION

Timely line outage detection and identification on power transmission networks is crucial for the secure and reliable operation of the system. There are many methods power system operators use to detect line outages [1]-[3]. However, many of these methods are not real-time and rely on a system model that is obtained offline, which can be inaccurate due to bad historical or telemetry data. Such inaccuracies have been a contributing factor in many recent blackouts. For example, in the 2011 San Diego blackout, operators were unable to determine overloaded lines because the network model was not up to date [4]. This lack of real-time situational awareness limited the ability of the operators to identify the contingency, contributing to a cascading failure. Similarly, during the 2003 US Northeastern blackout, operators failed to initiate the correct remedial schemes because they had an inaccurate model of the power system and could not identify the loss of key transmission elements [5]. These blackouts highlight the importance of developing online measurement-based techniques to detect and identify system topological changes that arise from line outages. This paper presents a method to detect and locate the faults on a line by utilizing measurements provided by phasor measurement units (PMUs). It extends the method proposed in [6] by providing the location of the line fault in addition to detection of the outage. The method is a two-step process in which the faulted-line is detected first, followed by the locationing of the actual fault. Case studies are performed on the IEEE 14-bus system to show successful detection and identification of line faults within in the distribution that results from the outage.

# II. POWER SYSTEM MODEL

This section introduces the power system model used for the line fault detection.

### A. Pre-fault Network

In a pre-fault network, the current injections at the buses of a power system is related to the bus voltage through

$$I[k] = Y[k]V[k], \tag{1}$$

where Y[k] is the system admittance matrix.



Fig. 1: Fault-on network.

## B. Fault-on Network

Suppose a line-to-ground fault occurs somewhere along line  $(j, \ell)$ , as shown in Fig. 1, with fault current  $I_{\rm f}$ . The fault divides line  $(j, \ell)$  into two sections, one with admittance  $y_{(j,\ell)}^j$  and the other with  $y_{(j,\ell)}^\ell$ . Referring to Fig. 1 for the fault-on circuit, Kirchhoff's Laws written for all nodes can be collected into

$$\begin{bmatrix} I[k] \\ I_{\rm f}[k] \end{bmatrix} = \begin{bmatrix} Y_{(j,\ell)} & \overline{y}_{(j,\ell)} \\ \overline{y}_{(j,\ell)}^{\rm T} & y_{(j,\ell)} \end{bmatrix} \begin{bmatrix} V[k] \\ V_{\rm f}[k] \end{bmatrix},$$
(2)

which contains one more node (to represent the fault location) than the pre-fault circuit described by (1). In (2),  $\overline{Y}_{(j,\ell)}$  is identical to Y, except in the following entries: i) along the main diagonal,  $\overline{Y}_{(j,\ell)}[j,j] = Y[j,j] - y_{(j,\ell)}^j$  and  $\overline{Y}_{(j,\ell)}[\ell,\ell] = Y[\ell,\ell] - y_{(j,\ell)}^\ell$ , and ii)  $\overline{Y}_{(j,\ell)}[j,\ell] = \overline{Y}_{(j,\ell)}[\ell,j] = Y_{(j,\ell)}[j,\ell] + y_{(j,\ell)}$ . Additionally,  $\overline{y}_{(j,\ell)} \in \mathbb{C}^N$  is a vector with the j entry equal to  $-y_{(j,\ell)}^j$ ,  $\ell$  entry equal to  $-y_{(j,\ell)}^\ell$ , and all other entries equal to 0.

From the last row of (2), we can solve for  $V_{\rm f}[k]$  as

$$V_{\rm f}[k] = \frac{1}{y_{(j,\ell)}} \left( I_{\rm f}[k] - \overline{y}_{(j,\ell)}^{\rm T} V[k] \right) \tag{3}$$

Then, substitute (3) into first row of (2) to get

$$I[k] = \overline{Y}_{(j,\ell)}V[k] + \frac{1}{y_{(j,\ell)}} \left( I_{\mathrm{f}}[k] - \overline{y}_{(j,\ell)}^{\mathrm{T}}V[k] \right) \overline{y}_{(j,\ell)}$$

$$\tag{4}$$

$$= \left(\overline{Y}_{(j,\ell)} - \frac{1}{y_{(j,\ell)}}\overline{y}_{(j,\ell)}^{\mathrm{T}}\overline{y}_{(j,\ell)}\right)V[k] + \frac{I_{\mathrm{f}}[k]}{y_{(j,\ell)}}\overline{y}_{(j,\ell)}$$
(5)

$$=:\widetilde{Y}_{(j,\ell)}V[k] + \frac{I_{\mathbf{f}}[k]}{y_{(j,\ell)}}\overline{y}_{(j,\ell)}.$$
(6)

Due to the structure of the problem, it is straightforward to show that  $\tilde{Y}_{(j,\ell)} = Y + \Delta Y_{(j,\ell)}$ , in which  $\Delta Y_{(j,\ell)}$  has only four nonzero entries. Furthermore, it can be expressed as the following rank-one matrix:

$$\Delta Y_{(j,\ell)} = \sigma_{(j,\ell)} r_{(j,\ell)} r_{(j,\ell)}^{\mathrm{T}},\tag{7}$$

where  $r_{(j,\ell)} \in \mathbb{R}^N$  is a vector with the *j* entry equal to 1,  $\ell$  entry equal to -1, and other all other entries equal to 0, and  $\sigma_{(j,\ell)} \in \mathbb{C}$  is a scalar expressed as

$$\sigma_{(j,\ell)} = -\frac{1}{y_{(j,\ell)}} \left( \left( y_{(j,\ell)}^j \right)^2 + y_{(j,\ell)}^j y_{(j,\ell)}^\ell + \left( y_{(j,\ell)}^\ell \right)^2 \right).$$
(8)

### C. Incremental Model

Denote the variations in the complex voltage at bus *i* to be  $\Delta V_i[k] \coloneqq V_i[k+1] - V_i[k]$  and the variations in the complex current at bus *i* to be  $\Delta I_i[k] \coloneqq I_i[k+1] - I_i[k]$ . Assume that the noise in V[k] is a Gaussian vector (i.e.,  $\Delta V[k] \sim \mathcal{N}(0, \Lambda)$ ), where  $\Lambda$  is the covariance matrix. Then, under pre-fault conditions, the noise in I[k] is also a Gaussian vector with  $\Delta I[k] \sim f_0 = \mathcal{N}(0, Y\Lambda Y^T)$ . When a fault occurs on line  $\ell$ in the system, the noise in I[k] is a Gaussian vector  $\Delta I[k] \sim f_{(j,\ell)} = \mathcal{N}(\frac{I_f[k]}{y_{(j,\ell)}}\overline{y}_{(j,\ell)}\Lambda \widetilde{Y}_{(j,\ell)}^T)$ .



Fig. 2: Sample paths for IEEE 14-bus system.

# **III. FAULT DETECTION AND IDENTIFICATION**

Following similar developments as [6], we compute  $L \times N$  CuSum statistics in parallel, where L is the number of lines in the system and N is the number of partitions for each line:

$$W_{(j,\ell)}[k] = \max\left\{\Omega_{(j,\ell)}^{(1)}[k], \dots, \Omega_{(j,\ell)}^{(T)}[k], \log\frac{f_{(j,\ell)}^{(0)}\Delta\hat{I}[k])}{f_0(\Delta\hat{I}[k])}\right\},\tag{9}$$

where

$$\Omega_{(j,\ell)}^{(i)}[k] = \max\left\{ \left[ \max\{\Omega_{(j,\ell)}^{(i)}[k-1], \Omega_{(j,\ell)}^{(i-1)}[k-1]\} + \log\frac{f_{(j,\ell)}^{(i)}(\Delta \hat{I}[k])}{f_0(\Delta \hat{I}[k])} \right], 0 \right\},$$
(10)

for  $i \in \{1, \ldots, T\}$ ,  $\Omega_{(j,\ell)}^{(0)}[k] \coloneqq 0$  for all  $k \in \mathbb{Z}$  and  $\Omega_{(j,\ell)}^{(i)}[0] \coloneqq 0$  for all  $\ell \in \mathcal{L}$  and all i and j.

A fault is declared when one of the streams crosses a predefined threshold A. In practice, the algorithm can be split into a two-stage process where the first stage computes the L streams for determining the line that experienced a fault and the second stage computes the N streams that determines the location of the line fault.

# **IV. CASE STUDIES**

The fault location and detection algorithm is applied to an IEEE 14-bus test system (the one line diagram for this system can be found at [7]). A single line-to-ground fault is simulated for line 2 with a fault location at the middle of the line. Figure 2 shows the resulting plots from executing the fault detection algorithm with A = 200. In Fig. 2(a), it can be observed that the stream for detectining a fault in line 2 crosses the threshold of 200 first, which means that the algorithm detects a fault in line 2. Figure 2(b) shows that for N = 5 (i.e., line 2 is partitioned into 5 sections), the stream for section 3 crosses the threshold of 200 first, which correctly identifies the section in which the fault occurred, which is at the middle of the line.

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